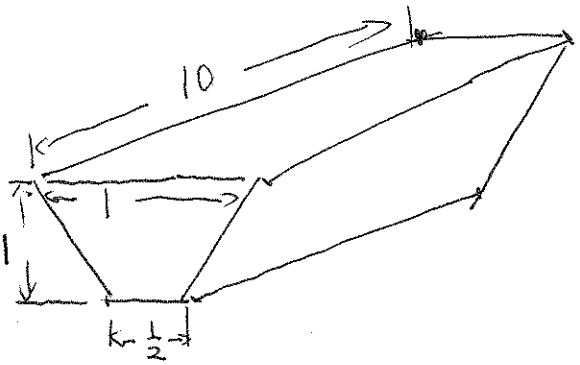
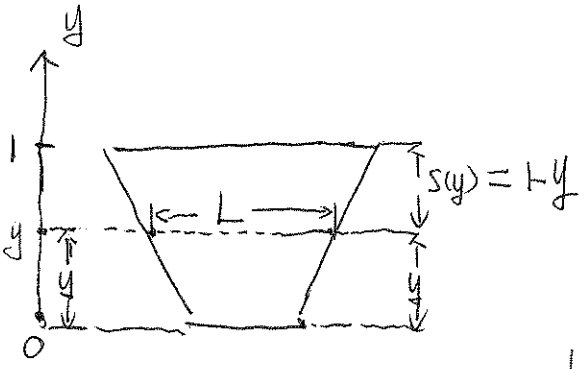
Extra Water Pumping Example from other Section

A cattle trough has a TRAPEZOIDAL cross section with height of 1 ft and horizontal sides of length $\frac{1}{2}$ ft and 1 ft. Assume the the length of the trough is 10 ft. the trough is filled with oil weighing 2 lb/ft^3. How much work is required to pump the oil out of the trough (to the level of the top of trough)?

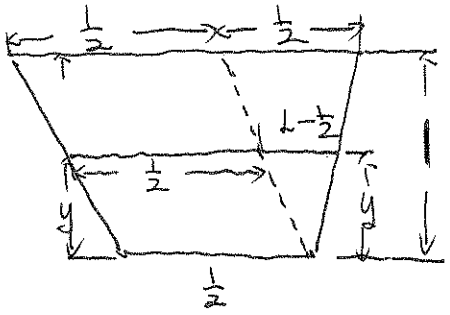




Horizontal cross section at height y is a rectangle

$$A(y) = L \times 10$$

↑ hard part is to find L through similar triangles.



$$\frac{L-\frac{1}{2}}{\frac{1}{2}} = \frac{y}{1} \Rightarrow L-\frac{1}{2} = \frac{1}{2}y \Rightarrow L = \frac{1}{2}y + \frac{1}{2}.$$

$$A(y) = \left(\frac{1}{2}y + \frac{1}{2}\right) \times 10 = \boxed{5y+5}.$$

$$W = \int_0^1 2 \cdot (1-y) \cdot (5y+5)\, dy$$

$$= \int_0^1 10 \cdot (1-y) \cdot (y+1) \cdot dy$$

$$= \int_0^1 10(1-y^2)\, dy = 10 \cdot \left(y - \frac{1}{3}y^3\right)\Big|_0^1 = 10 \cdot \left(1 - \frac{1}{3}\right) = \boxed{\frac{20}{3} \text{ ft-lb}}$$